THE CALCULATION OF THE SPRING CONSTANT AND EXPERIMENTAL VALIDATION

MICHAŁ SZULEWSKI[1)]

[1)] michal.szulewski@caetica.com

Abstract:

The article presents calculations of the tension spring constant and its validation performed on the test stand designed and built up by Caetica engineering consultancy caetica.com. The goal of the study was to obtain reliable stiffness data of the spring to use it in further calculations and in FEM (Finite Element Method) simulations.

Keywords: spring, constant, stiffness, validation, engineering consultancy Caetica, FEA, FEM

1. Introduction

Springs are often used structural components in industrial machines and also in daily used constructions at home – e.g. garden swings.

Main functions of spring are exerting a force, accumulating and dissipating an energy. Due to the method of loading springs can be divided into 4 main types: tension, compression, torsion and bending. There are also divisions according to the pitch of the winding, the shape of the winding and others.

Force generated by a linear spring can be expressed as follow:

$$F = -k\Delta x \qquad (1)$$

where:

$F - force\ (N)$
$k - spring\ constant\ (N/mm)$
$x - displacement\ (mm)$

After transforming equation 1. it can be noticed that spring constant is a ratio of force generated in a spring to its elongation. It can be written as below:

$$k = \frac{F}{\Delta x} \qquad (2)$$

Above equation 2. describes generic case. After taking into account spring's geometry and its material, equation 2. can be formulated as below:

$$k = \frac{F}{\Delta x} = \frac{Gd^4}{8D^3n} \qquad (3)[1]$$

where:

$G - shear\ modulus\ (MPa)$
$d - wire\ diameter\ (mm)$
$D - mean\ diameter\ (mm)$
$n - number\ of\ working\ windings$

2. The goal and assumptions

The goal of the study was to determine the spring constant k by hand calculations and prove the results by experimental validation. Tension spring presented in the picture below has been examined. Material of the spring – stainless steel $G = 80769MPa$

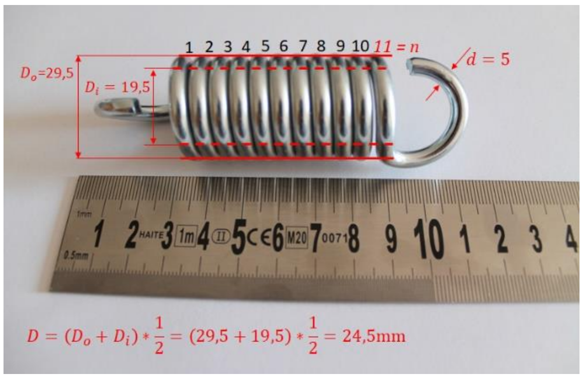

Fig. 1 The tension spring used in the study

To achieve the goal a test stand has been design and built. The test stand is based on a pneumatic actuator powered up by a compressor. Force is measured by a digital crane scale. Displacement is measured by a dial indicator. Following figures 2-3 present the test stand.

Fig. 2 The test stand – the spring with measurement devices: the crane scale and the dial indicator

The assumption was to build a test stand with an optimal ratio of measurement accuracy to the stand cost. An ease of the machine modification has been also taken into account.

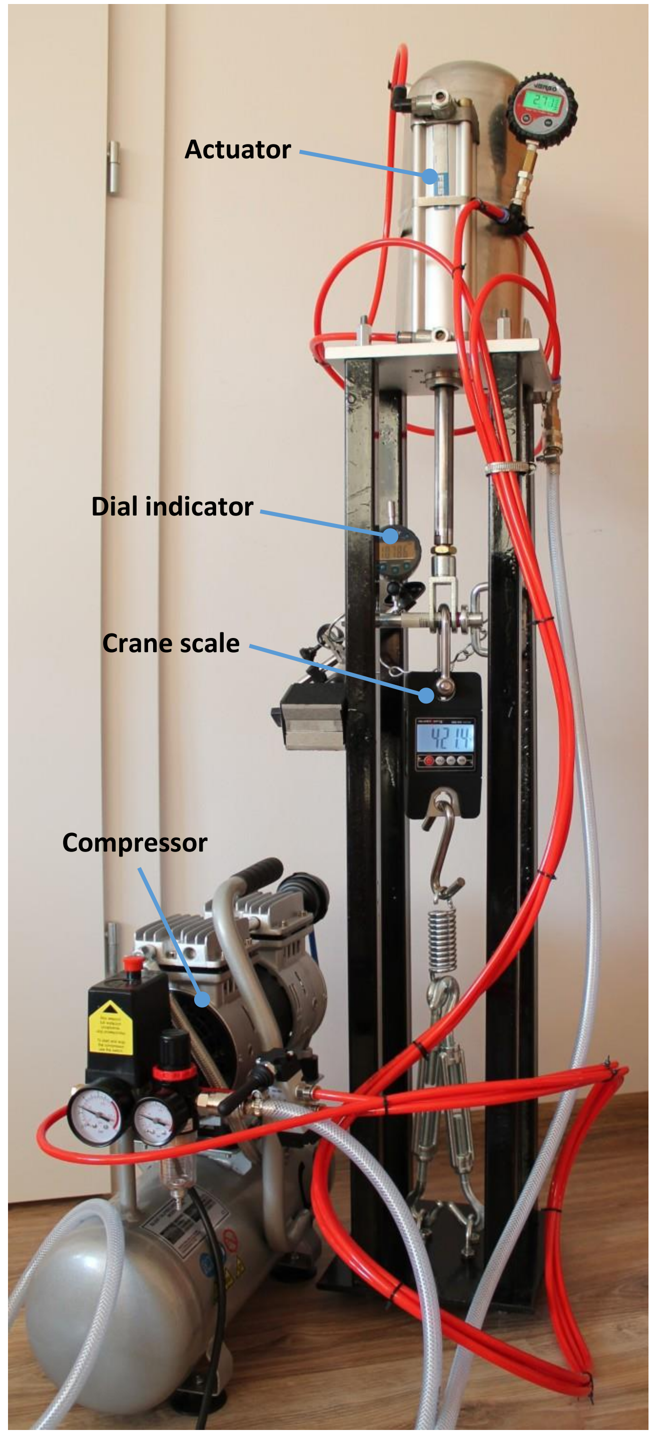

Fig. 3 The test stand - global view

3. Results and discussion

Theory

After substituting the geometrical data of the spring (fig.1) to the formula in equation 3.,the following is obtained:

$$\boldsymbol{k} = \frac{Gd^4}{8D^3n} = \frac{\frac{80769N}{mm^2} * (5mm)^4}{8 * (24{,}5mm)^3 * 11} = \frac{\mathbf{39.0N}}{\boldsymbol{mm}} \quad (4)$$

Above result k=39N/mm can be interpreted: the spring elongates 1mm under 39N of force.

Experiment

Seven measurements of force and displacement have been done to create characteristics. The measurement was done in 0 to 8.3mm displacement range of the spring. The data has been collected in chart in figure 4. Linear approximation has been used to obtain the slope factor of the curve which corresponds to the spring constant. Measurement uncertainty has been skipped.

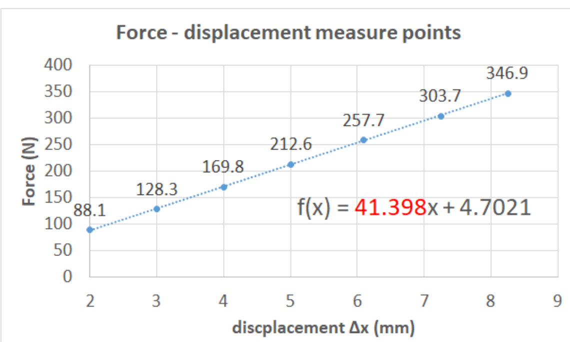

Fig. 4 The experimental measurement - force and displacement

From linear approximation (fig.4) of data points, linear function has been obtained:

$$Force = 41.398x + 4.7021 \quad (5)$$

$$\boldsymbol{k_{experimental}} = \mathbf{41.4}\frac{\boldsymbol{N}}{\boldsymbol{mm}}$$

Intercept constant 4.7021 in equation 5. should be equal to 0 in theoretical approach. In the performed measurement intercept constant >0 could be caused by too high initial tension in crane scale or not tared device.

Relative error δ has been calculated to evaluate results accuracy :

$$\delta = \frac{\Delta x}{x} = \frac{|x - x_0|}{x} = \frac{|39{,}0 - 41{,}4|}{39{,}0} = 6.2\%$$

4. Summary

Results obtained in the experiment measurement in subjective opinion can be considered satisfactory taking into account the project assumption - optimal price accuracy ratio. Stiffness data can be used to further analysis e.g. Finite Element Analysis.

References:

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